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MIMO RADAR WAVEFORM COMPARISON USING AMBIGUITY FUNCTION

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ABSTRACT

Ambiguity function provides an effective way for analysis of the resolving performance of a radar system. Multiple-Input Multiple-Output radar has been shown to provide greater performance in theory and in practice. MIMO radars are equipped with the capacity to freely choose their transmitted waveforms at each aperture. In conventional radar systems Woodward's ambiguity function is used to characterize waveform resolution performance. In this paper comparison of results is done by plotting the waveform for multiple-input-multiple-output radar ambiguity function for various inputs signals.

KEYWORDS: Radar Waveform, Ambiguity Function (AF), Multiple-Input Multiple-Output (MIMO)

INTRODUCTION

Radar is an electromagnetic sensor for the detection and location of reflecting objects. The radar radiates electromagnetic energy from an antenna to propagate in space. Some of the radiated energy is intercepted by a reflecting object usually called a target located at a distance from the radar. The energy intercepted by the target is reradiated in many directions. Some of the reradiated (echo) energy is returned to and received by the radar antenna.[1] After amplification by a receiver and with the aid of proper signal processing a decision is made at the output of the receiver as to whether or not a target echo signal is present. At that time the target location and possibly other information about the target is acquired.

Radar application are military application, remote sensing of the environment, Air traffic control, Law enforcement and highway security, Aircraft safety and navigation, Ship safety and Space.

Multiple-input multiple-output (MIMO) systems have gained popularity and attracted attention for their ability to enhance all areas of system performance. MIMO ideas are not new, in fact their origin can be traced to the control systems. The idea of optimally selecting multiple system inputs to enhance parameter estimation. The early 1990s saw an emergence of MIMO ideas into the field of communication systems. More recently, one will find the ideas of MIMO appearing in sensor and radar systems.

A MIMO radar system consists of transmit and receive sensors [2]; the transmit sensors have the ability to transmit arbitrary and independent waveforms [3]. In many ways a MIMO radar is similar to a MIMO communication system. Although the mission of a radar system is quite different, among the many possible uses of a radar system, tracking and detecting targets, estimating target model parameters, and creating images of targets are some of the most common[4].

The radar ambiguity function represents the output of the matched filter, and it describes the interference caused by range and/or Doppler of a target when compared to a reference target of equal Radar Cross Section. The ambiguity

function evaluated at is equal to the matched filter output that is matched perfectly to the signal reflected from the target of interest. In other words, returns from the nominal target are located at the origin of the ambiguity function. Thus, the ambiguity function at nonzero and represents returns from some range and Doppler different from those for the nominal target.

Advantages of MIMO Radar: Better detection performance, Better angular resolution, Better angular measurement accuracy Improved robustness against Radar Frequency Interferometer (RFI) & electronic counter measure (ECM) & multipath. The radar ambiguity function is normally used by radar designers as a means of studying different waveforms. It can provide understanding about how different radar waveforms may be used for the various radar applications[5]. It is also used to determine the range and Doppler resolutions for a specific radar waveform.

The paper is organized as follows: first section gives the overview of AF developed by Woodward and gives background for establishing MIMO AF. Based on this waveform are compared for MIMO AF. Afterwards, various radar waveforms for MIMO AF are simulated and results are compared.

AMBIGUITY FUNCTION

The ambiguity function is "the squared magnitude $|\chi(\tau,\omega)|^2$ of the function that describes the response of a radar receiver to targets displaced in range (time delay, τ) and doppler frequency, f_d, from a reference position, where the function $|\chi(0,0)|^2$ is normalized to unity. Mathematically,

$$\chi(\tau,\omega) = \int_{-\infty}^{\infty} u(t)u^*(t+t_d)\exp(j2\pi f_d t)dt \tag{1}$$

Where $u(t)$ is the transmitted envelope waveform, suitably normalized, positive t indicates a target beyond the reference delay, and positive f_d indicates an incoming target. Used to examine the suitability of radar waveforms for achieving accuracy, resolution, freedom from ambiguities, and reduction of unwanted clutter." The function was first introduced by Woodward. [1]

Invariant Relation for the Cross-Ambiguity Function

The above defined relation is for *auto-ambiguity function*. If *cross-ambiguity function* is defined for two complex waveform $u_1(t)$ and $u_2(t)$ and both signal are normalized then result identical to equation (1) can be shown to exist whose function $\chi_{12}(\tau,\omega)$ is defined as

$$\chi_{12}(\tau,\omega) = \int_{-\infty}^{\infty} u_1(t)u_2^*(t+t_d)\exp(j2\pi f_d t)dt \tag{2}$$

In such systems it is desirable to have waveforms which can be identified correctly, with high probability, despite time shifts and frequency (Doppler) shifts which may not be known at the receiver. Thus the distribution of this combined correlation function of waveforms over the time-frequency plane is of interest [6].

MIMO AMBIGUITY FUNCTION

Drawback of AF introduced by Woodward's ambiguity function needs modification to handle larger bandwidth signals, long duration signals, and targets with high velocity therefore need for MIMO AF was required.

The Ambiguity Function of Monostatic and Bistatic Radar

The ambiguity function is widely used as an important tool for evaluating the performance of a monostatic radar waveform. Woodward [3] has indicated its form:

$$\Theta_{mono} = |X(\tau,\omega_D)|^2 = \left| \int_{\infty}^{+\infty} \tilde{s}(t)\tilde{s}^*(t-\tau)e^{j\omega_D t}dt \right|^2 \tag{3}$$

Where τ and ω_D are time and frequency shifts of received signal $X(\tau,\omega_D)$ is auto-correlation function of complex modulation $\tilde{s}(t)$ and is output of matched filter.The AF for bistastic radar has been developed in [7]. The receiver and the transmitter are spatially separated. If this geometryis considered, the bistatic ambiguity function will have the same form as in equation (1), but the time and Doppler shift will depend on the baseline length L, the angle θ_R, the range R_R and the velocity $Vcos\phi$[7], in a non-linear way. Thus, implementing equation (3), the bistatic ambiguity function will be influenced by the transmitter target receiver geometry.

MIMO radar can be treated as MN bistatic radars. If θ_R, R_R and $Vcos\phi$ are selected as variables in north-reference coordinate system, then different bistatic radar has different θ_R, R_R and $Vcos\phi$ values, that is, the ambiguity function has multiple peaks[8]. In order to overcome this disadvantage, right-angle coordinate system is selected as reference system and the target position and velocity are selected as the variables of the ambiguity function.

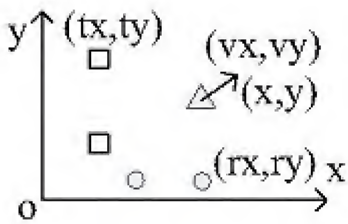

Figure 1: Right-Angle Coordinate System

Assume all transmitters-target-receivers are in the sameplane *xoy* in figure 1. The plane denotes transmitters, the circle denotes receivers and the triangle denotes target. The coordinates of M transmitters are(tx_i, ty_i),i =1,…, M, The coordinates of N receivers are (rx_j, ry_j), $j=1,...,N$, target in the plane moves in the velocity of (v_x,vy).Assuming pulse is transmitted at t = 0, pulse arrived at target at $t = \tau_1$and pulse is received at $t = \tau_a$.In the momentof t = 0,the position oftargetis $(x(0),y(0))$, then the track of the target can be expressed as $x(t)=x(0)+v_xt$, $y(t)=y(0)+v_yt$.Using the ith transmitter-target-jth receiver building up a bistatic radar. Assuming x=x(τ_1), y=y(τ_1).The ambiguity function of bistatic radar in right-angle coordinate system

The ambiguity function in right-angle coordinate system is,

$$\Theta_{bi_{jk}} =$$

$$\left| \int_{\infty}^{+\infty} \tilde{s}(t)\tilde{s}^*(t-\tau(x,y))e^{j\omega_D(x,y,v_x,v_y).t}dt \right|^2 \tag{4}$$

The Ambiguity Function of MIMO Radar

In MIMO radar, the transmitted signals are required to be mutually orthogonal. So at each of the receiver, the received signals are matched filtered for each of the transmitted waveforms forming MN channels [9]. Then the ambiguity function of MIMO radar is a combination of the ambiguity function of bistatic radar [10].

$$\Theta_{mimo} = \sum_{i=1}^{M}\sum_{j=1}^{N}\Theta_{bi_{ij}} = \sum_{i=1}^{M}\sum_{j=1}^{N}\left|X_{ij}\right|^2 \tag{5}$$

DIFFERENT TYPES OF SIGNAL

Single Frequency Pulse Ambiguity Function

This is also called as constant frequency pulse or unmodulated pulse is basic radar signal. MIMO AF is developed for this signal which is defined by

$$s(t) = \frac{1}{\sqrt{\tau'}} Rect\left(\frac{t}{\tau'}\right) \tag{6}$$

Where τ' is pulse width. Substituting in equation (3) following equation for mono ambiguity function is obtained

$$|\chi(\tau,\upsilon)|^2 =$$

$$\left|\left(1-\frac{|\tau|}{\tau'}\right)\frac{\sin\left(\pi\upsilon\left(\tau'-|\tau|\right)\right)}{\pi\upsilon\left(\tau'-|\tau|\right)}\right|^2, |\tau| \le \tau' \tag{7}$$

Delay cut in AF is obtained by making $\upsilon = 0$ and Doppler cut is obtained by making $\tau = 0$.

LFM Ambiguity Function

Linear frequency modulation (LFM) is the first and probably still is the most popular pulse compression method[11].The Linear Frequency Modulation (LFM) complex envelope signal is defined by

$$s(t) = \frac{1}{\sqrt{\tau'}} Rect\left(\frac{t}{\tau'}\right) e^{j\pi\mu t^2} \tag{8}$$

Where μ is frequency slope which is ratio of sweep frequency band B and pulse duration τ' .The AF for LFM signal after substituting in equation (3) is

$$|\chi(\tau,\upsilon)|^2 = \left|\left(1-\frac{|\tau|}{\tau'}\right)\frac{\sin\left(\pi\tau'(\mu\tau+\upsilon)\left(1-\frac{|\tau|}{\tau'}\right)\right)}{\pi\tau'(\mu\tau+\upsilon)\left(1-\frac{|\tau|}{\tau'}\right)}\right|^2 , \tau \le \tau' \tag{9}$$

Stepped FM Ambiguity Function

A stepped frequency pulse waveform consists of a series of N narrowband pulses. The frequency is increased from step to step by a fixed amount, Δf, in Hz. Similar to linear FM pulse waveforms, stepped frequency waveforms are a

popular pulse compression technique. Using this approach enables you to increase the range resolution of the radar without sacrificing target detection capability. Its complex envelop is defined as

$$s(t)=\frac{1}{\sqrt{N}}e^{j\pi\mu_N t^2}\sum_{n=0}^{N-1}s_p(\mathrm{t-nt}_r), \tag{10}$$

Where s_p is LFM equation (7) for duration $t_p$and t_r is time repetition interval and is choosen such that t_r/t_p is greater than 2 and frequency slope μ and $\mu_N$are positive.

Ambiguity Function for Barker Code

A Barker code or Barker sequence is a finite sequence of N values of +1 and −1, the waveforms consists of a concatenation of short pulses (termed subpulses) where the phase, $\phi(t)$, is constant over the duration of each subpulse but varies from subpulse-to-subpulse. An equation for the waveform can be written as

$$s(t)=\sum_{k=0}^{N-1}e^{j\phi_k}\,\mathrm{rect}\left[\frac{t-k\tau_{sp}}{\tau_{sp}}\right] \tag{11}$$

Where the ϕ_k are the phases of the individual subpulses, τ_{sp} is the width of the individual subpulses and N is the number of subpulses that make up the overall pulse. The overall pulse width, τ_p is $\tau_p = N\tau_{sp}$.

Ambiguity Function for Frank Code

The Frank code is derived from a step approximation to a linear frequency modulation waveform using N frequency steps and N samples per frequency. Hence the length of Frank code is N^2. The Frank coded waveform consists of a constant amplitude signal whose carrier frequency is modulated by the phases of the Frank code.

The phases of the Frank code is obtained by multiplying the elements of the matrix A by phase (2π/N) and by transmitting the phases of row1 followed by row 2 and so on.

$$A=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 2 & \cdots & (\mathrm{N}-1) \\ 0 & 2 & 4 & \cdots & 2(N-1) \\ 0 & 3 & 6 & \cdots & 3(N-1) \\ & & & & \\ 0 & (N-1) & 2(N-1) & & (N-1)^2 \end{bmatrix} \tag{12}$$

The phase of *i*th code element in the *j*th row of code group is computed as

$$\Phi_{i,j}=\left(\frac{2\pi}{N}\right)(\mathrm{i}-1)(\mathrm{j}-1) \tag{13}$$

RESULTS

The $|\chi(\tau,\upsilon)|$ plots are characterized by special names for specific values of τ and υ . If value of $\upsilon=0$ then $|\chi(\tau,0)|$ is said to be matched in Doppler, Delay-cut of the ambiguity function. Which is output of the classical

matched filter. If value of $\tau = 0$ then $|\chi(0,\upsilon)|$ is said to be matched in range, Doppler cut of the ambiguity function.

Single frequency pulse waveforms are generated and MIMO AF is formed for 4x4, four plot are generated 3d plot, contour, delay cut and doppler cut plotted in figure 1 and 2.

Linear frequency modulation waveforms are generated and MIMO AF is formed for 4x4, four plot are generated 3d plot, contour, delay cut and doppler cut plotted in figure 3 and 4.

Stepped FM waveforms are generated and MIMO AF is formed for 4x4, four plot are generated 3d plot, contour, delay cut and doppler cut plotted in figure 5 and 6.

Barker Code are generated and MIMO AF is formed for 4x4, four plot are generated 3d plot, contour, delay cut and doppler cut plotted in figure 7 and 8.

Frank Code are generated and MIMO AF is formed for 4x4, four plot are generated 3d plot, contour, delay cut and doppler cut plotted in figure 9 and 10.

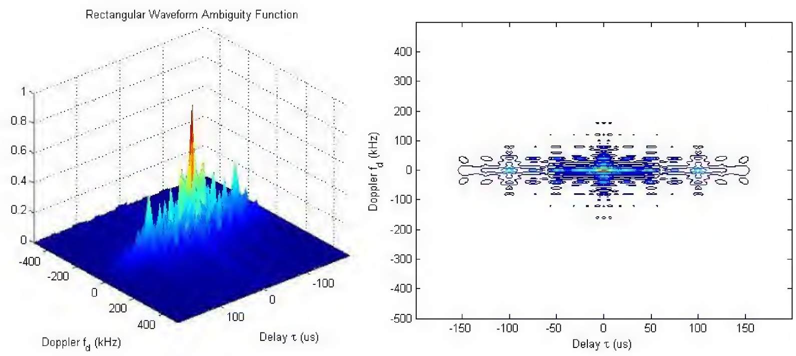

Figure 2: 3d Plot for 4x4 MIMO and Contour Plot for Single Frequency Pulse

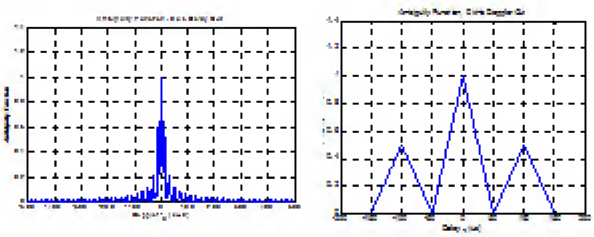

Figure 3: Delay Cut for 4x4 MIMO and Doppler Plot for Single Frequency Pulse

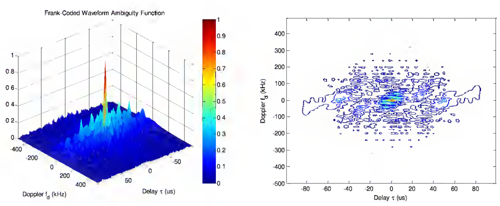

Figure 4: 3d Plot for 4x4 MIMO and Contour Plot for Linear Frequency Modulation Waveform

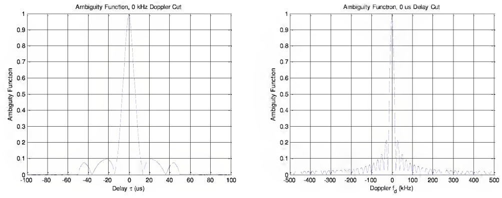

Figure 5: Delay Cut for 4x4 MIMO and Doppler Plot for Linear Frequency Modulation

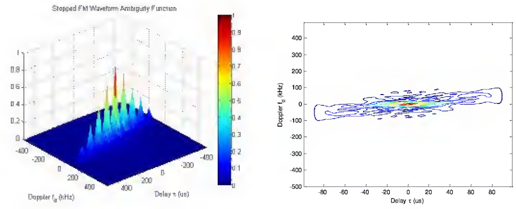

Figure 6: 3d Plot for 4x4 MIMO and Contour Plot for Stepped Frequency Modulation

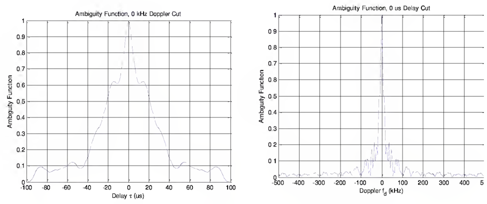

Figure 7: Delay Cut for 4x4 MIMO and Doppler Plot for Stepped Frequency Modulation

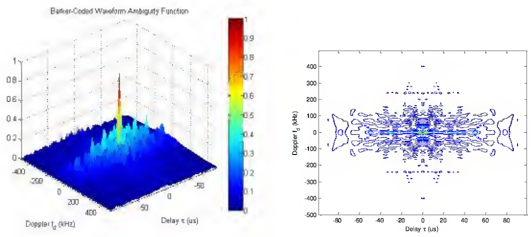

Figure 8: 3d Plot for 4x4 MIMO and Contour Plot for Barker Code Waveform

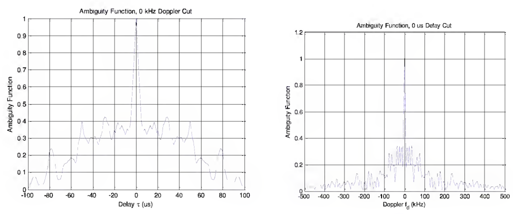

Figure 9: Delay Cut for 4x4 MIMO and Doppler Plot for Barker Code Waveform

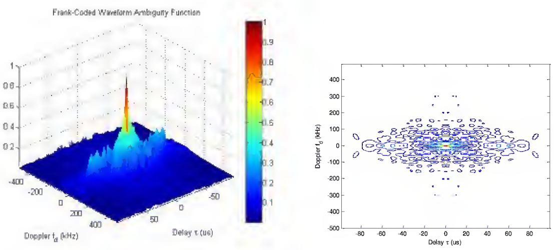

Figure 10: 3d Plot for 4x4 MIMO and Contour Plot for Frank Code Waveform

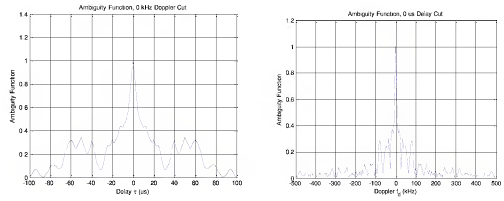

Figure 11: Delay Cut for 4x4 MIMO and Doppler Plot for Same

Comparison results as shown below for Delay width and Doppler cut

Table 1: Comparison Results of Various Waveforms for Delay Cut

Description	Rectangular Pulse	LFM	Stepped FM	Barker Code	Frank Code
ain lobe Width(KHz)	6	24	17	11.01	13.82
Number of Sidelobes(pair)	25	25	6	26	19
Main lobe to side lobe ratio	1.539	4.6926	4.721	2.943	2.704

Table 2: Comparison Results of Various Waveforms for Doppler Cut

Description	Rectangular Pulse	LFM	Stepped FM	Barker Code	Frank Code
Main lobe Width(us)	50	13	11.2	10	9.2
Number of Sidelobes(pairs)	1	2	6	6	6
Main lobe to side lobe ratio	2	10.34	2.367	2.28	2.518

Delay-cut plot null gives idea of how two targets can be resolved in terms of doppler-frequency(f_d) from first null which is converted to velocity(v) by using relation $v=f_d*c/f_c$. Where c=velocity of light and $f_c$is carrier frequency.SimilarlyDoppler-cut plot null gives the idea of how two targets can be resolved in terms of time(t) from first null which is converted to range(R) by using basic relation of velocity(v=R/t).

Table 1 shows that rectangular has high ability and LFM has least ability to resolve in terms of frequency but wastage of energy is more in former one because of larger sidelobes.

Table 2 gives the idea that frank code has higher ability to resolve and rectangular pulse low ability to resolve when targets are placed closer, but in terms of energy consumption LFM is the best due to high main lobe to sidelobe ratio.

CONCLUSIONS

AF is efficient tool used for resolving performance of radar system. In this paper a comprehensive study of AF in MIMO radar. It shows how monostatic AF is used to form MIMO AF.

Results presented give the idea of how different types of waveforms are obtainedusing various inputs for MIMO AF.

Delay-cut helps in resolving two different target if the difference between velocities is more that main lobe width and Doppler-cut helps in resolving two different target if the difference between ranges is more that main lobe width.

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